Name:_____ **Solutions** _____

> This assignment consists of seven questions, each worth five points for a total of 35 points. To receive full credit you must **show all necessary work**. You should write your answers in the spaces provided, but if you require more space please *staple any extra sheets* you use to this assignment. If you are having trouble with any of the problems, look at the lecture notes and exercises in the lecture notes for help. Remember to start this assignment early, your next quiz is based on this assignment.

concave up   up | down
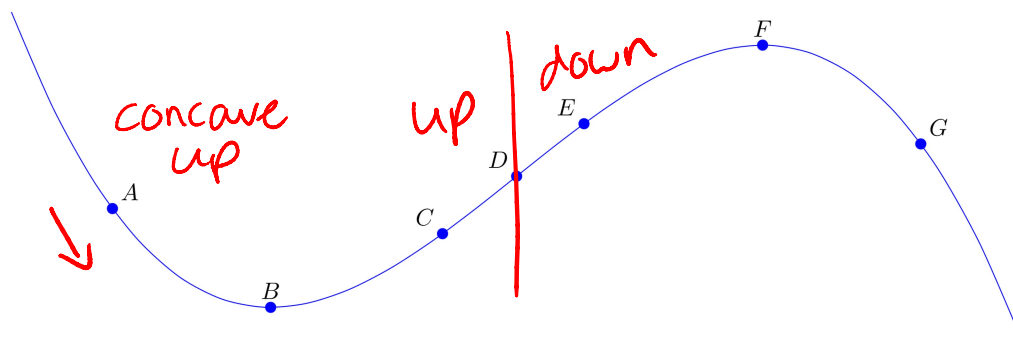
1. Consider the graph of the function $f(x)$ above. Determine if the following quantities are positive, negative or zero.

   (a) The derivative at A

   Answer:___ negative ___

   (b) The second derivative at A

   Answer:___ positive ___

   (c) The second derivative at D

   Answer:___ zero ___

2. Consider the graph of the function $f(x)$ above. Find the point that matches the following descriptions.

   (a) $f'(x) > 0$, $f''(x) < 0$

   Answer:___ E ___

   (b) $f'(x) > 0$, $f''(x) > 0$

   Answer:___ C ___

   (c) $f'(x) < 0$, $f''(x) < 0$

   Answer:___ G ___

3. Let $C(q)$ represent the total cost, in dollars, of producing q items. Suppose $C(15) = 2,300$ and $C'(15) = 108$. Estimate the total cost of producing

   (a) 16 items

   $$C(16) \approx C(15) + C'(15)$$

   Answer: $\underline{\ \ C(16) \approx 2,408 \ \ }$

   (b) 14 items

   $$C(14) \approx C(15) - C'(15)$$

   Answer: $\underline{\ \ C(14) \approx 2,192 \ \ }$

4. To produce $1,000$ items, the total cost is $\$5,000$ and the marginal cost is $\$25$ per item. Estimate the costs of producing

   (a) $1,001$ items

   $$C(1001) \approx C(1000) + MC(1000)$$

   Answer: $\underline{\ \ C(1001) \approx 5025 \ \ }$

   (b) 999 items

   $$C(999) \approx C(1000) - MC(1000)$$

   Answer: $\underline{\ \ C(999) \approx 4975 \ \ }$

   (c) $1,100$ items

   $$C(1100) \approx C(1000) + 100 \, MC(1000)$$

   Answer: $\underline{\ \ C(1100) \approx 7,500 \ \ }$

   (d) 900 items

   $$C(900) \approx C(1000) - 100 \, MC(1000)$$

   Answer: $\underline{\ \ C(900) \approx 2,500 \ \ }$

5. An industrial production process costs $C(q)$ million dollars to produce q million units; these units then sell for $R(q)$ million dollars. If $C(2.1) = 5.1$, $R(2.1) = 6.9$, $MC(2.1) = 0.6$ and $MR(2.1) = 0.7$, calculate

(a) The profit earned by producing 2.1 million units

$$\pi = R - C = 6.9 - 5.1$$

Answer: $\$1.8$ million

(b) The approximate change in revenue if the production increases from 2.1 to 2.4 million units

$$R(2.4) \approx R(2.1) + 0.3\,MR(2.1) \quad \text{Change} = 0.3 \times 0.7$$

Answer: $\$0.21$ million

(c) The approximate change in revenue if the production decreases from 2.1 to 2.05 million units

$$R(2.05) \approx R(2.1) - 0.05\,MR(2.1) \quad \text{Change} = -0.05 \times 0.7$$

Answer: $-\$0.035$ million

(d) The approximate change in profit if the production increases from 2.1 to 2.4 million units

$$C(2.4) \approx C(2.1) + 0.3\,MC(2.1) \quad \text{Change in Cost} = 0.3 \times 0.6 = 0.18$$

$$0.21 - 0.18$$

Answer: $\$0.03$ million

(e) The approximate change in profit if the production decreases from 2.1 to 2.05 million units

$$C(2.05) \approx C(2.1) - 0.05\,MC(2.1) \quad \text{Change in Cost} = -0.05 \times 0.6 = -0.03$$

$$-0.035 - (-0.03)$$

Answer: $-\$0.005$ million

6. Differentiate

   (a) $A(t) = 3t^5$

   Answer: $A'(t) = 15t^4$

   (b) $r(p) = p^5 + p^3$

   Answer: $r'(p) = 5p^4 + 3p^2$

   (c) $f(x) = 5x^2 - 7x^3$

   Answer: $f'(x) = 10x - 21x^2$

   (d) $g(t) = \frac{t^2}{4} + 3$

   Answer: $g'(t) = \frac{1}{2}t$

7. The revenue, in dollars, from producing q units of a product is given by
$$R(q) = 1000q - 3q^2.$$

   (a) Find $R(125)$

   $R(125) = 1000(125) - 3(125)^2$

   Answer: $R(125) = 78125$

   (b) Find $R'(125)$

   $R'(q) = 1000 - 6q$
   $R'(125) = 1000 - 6(125)$

   Answer: $R'(125) = 250$

   (c) Determine if $R(q)$ is concave up or down at $q = 125$

   $R''(q) = -6$
   $R''(125) = -6$

   Answer: Concave down

The End.